Divisibility by 11

Rule. *A three-digit number is divisible by 11 if the sum of the outer two digits equals the value of the inner digit (modulo 11).*

Proof. Let $n = 100a + 10b + c$ where a, b, c are the three digits of n, $100 <= n <= 999$. We want to show that n is divisible by 11 if and only if $a + c \equiv b \pmod{11}$.

Suppose $a + c \equiv b \pmod{11}$ then we can write

$$\begin{aligned} n &= 100a + 10b + c \\ &= 100a + 10(a + c) + c \\ &= 110a + 11c \\ &\equiv 0 \pmod{11}, \quad \text{since } 11 \mid 110 \text{ and } 11 \mid 11. \end{aligned}$$

Thus $a + c \equiv b \pmod{11} \implies 11 \mid n$.

Conversely, suppose $11 \mid n$ then we have

$$\begin{aligned} 100a + 10b + c &\equiv 0 \pmod{11} \\ a - b + c &\equiv 0 \pmod{11}, \quad \text{since } 100 \equiv 1 \text{ and } 10 \equiv -1 \pmod{11} \\ a + c &\equiv b \pmod{11} \end{aligned}$$

Thus $11 \mid n \implies a + c \equiv b \pmod{11}$.

Hence we have the required result, $a + c \equiv b \pmod{11} \iff 11 \mid n$. □

Examples

$11 \mid 132$ since $1 + 2 = 3$
$11 \mid 209$ since $2 + 9 = 11 \equiv 0 \pmod{11}$