Divisibility by 3

Rule. *A number is divisible by 3 if and only if the sum of its digits is divisible by 3.*

Proof. Any number of arbitrary k decimal digits can be written in the form

$$n = d_1 + 10d_2 + 100d_3 + \ldots + 10^{k-1}d_k$$

where $d_1, d_2, \ldots, d_k$ are the decimal digits of n from right to left.

Since $10 \equiv 1 \pmod 3$ it follows that any power of 10 is also congruent to 1 modulo 3 and so we have the identity

$$n = d_1 + 10d_2 + 100d_3 + \ldots + 10^{k-1}d_k \equiv d_1 + d_2 + d_3 + \ldots + d_k \pmod 3$$

If the LHS (n) is congruent to 0 modulo 3 then so is the RHS (the sum of its digits) and vice versa.

It immediately follows that a number n is divisible by 3 if and only if the sum of its digits is divisible by 3. $\square$

This procedure is recursive and can be repeated until the result is small enough to spot that it is divisible by 3 (or not).

Examples

$3 \mid 123$ since $1 + 2 + 3 = 6$ and $3 \mid 6$
$3 \mid 987$ since $9 + 8 + 7 = 24$ and $3 \mid 24$ (since $2 + 4 = 6$ and $3 \mid 6$)
$3 \mid 17946$ since $1 + 7 + 9 + 4 + 6 = 27$ and $3 \mid 27$ (since $2 + 7 = 9$ and $3 \mid 9$)

The last two examples demonstrate the recursion process (for those of you who don't know your 3 times table!)